Third Semester B.E. Degree Examination, Feb./Mar. 2022

Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the modulus and amplitude of the complex number : $\dfrac{(2-3i)(2+i)^2}{1+i}$. **(07 Marks)**

b. Prove that $\left(\dfrac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n=\cos n\theta+i\sin n\theta$. **(06 Marks)**

c. Show that the vectors $\vec{a}-2\vec{b}+3\vec{c},\ -2\vec{a}+3\vec{b}-4\vec{c},\ -\vec{b}+2\vec{c}$ are coplanar. **(07 Marks)**

OR

2 a. Given $\vec{a}=2\hat{i}+2\hat{j}-\hat{k},\ \vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$. Find : i) $\vec{a}\cdot\vec{b}$ ii) $\vec{a}\times\vec{b}$ iii) $|\vec{a}\times\vec{b}|$. **(07 Marks)**

b. Determine the value of λ, so that $\vec{a}=2\hat{i}+\lambda\hat{j}-\hat{k}$, and $\vec{b}=4\hat{i}-2\hat{j}-2\hat{k}$, are perpendicular. **(06 Marks)**

c. Express $1-i\sqrt{3}$ in the polar form and hence find its modulus and amplitude. **(07 Marks)**

Module-2

3 a. Using Euler's theorem, prove that $xu_x+yu_y=-3\cot u$ where $u=\sin^{-1}\left(\dfrac{x^2y^2}{x+y}\right)$. **(07 Marks)**

b. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x}=1+x-\dfrac{x^2}{2}-\dfrac{x^3}{3}+\dfrac{x^4}{24}+\ldots$. **(06 Marks)**

c. If $u=x+3y^2,\ v=4x^2yz,\ w=2z^2-xy$, evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point $(1,-1,0)$. **(07 Marks)**

OR

4 a. Obtain Maclaurin's series expansion for the function e^x upto x^4. **(07 Marks)**

b. If $u=\sin^{-1}\left[\dfrac{x^3+y^3}{x+y}\right]$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2\tan u$. **(06 Marks)**

c. If $u=f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $x=(1-t^3),\ y=(1+t^2),\ z=(2t-5)$ determine its velocity and acceleration at $t=1$ sec. **(07 Marks)**

b. If $\vec{F}=2x^2\hat{i}-3yz\hat{j}+xz^2\hat{k}$, and $\phi=2z-x^3y$, find $\vec{F}\cdot(\nabla\phi)$ and $\vec{F}\times(\nabla\phi)$ at $(1,-1,1)$. **(06 Marks)**

c. Find the constants a, b, c so that $\vec{f}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational. **(07 Marks)**

OR

6 a. Find the directional derivate of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ along $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ (07 Marks)

b. Find curl $\vec{f}$ given that $\vec{f} = xyz^2\,\hat{i} + xy^2z\,\hat{j} + x^2yz\,\hat{k}$. (06 Marks)

c. If $\vec{f} = x^2i + y^2j + z^2k$ and $\vec{g} = yzi + zxj + xyk$. Show that $\vec{f} \times \vec{g}$ is a solenoidal vector. (07 Marks)

Module-4

7 a. Obtain the reduction formula, $I_n = \int \cos^n x\,dx$, where n is a positive integer. (07 Marks)

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x + y + z)\,dx\,dy\,dz$. (07 Marks)

OR

8 a. Evaluate : $\int_0^{\pi/6} \sin^6(3x)\,dx$. (07 Marks)

b. Evaluate : $\int_0^{\pi} x \sin^4 x \cos^6 x\,dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz\,dx\,dy\,dz$. (07 Marks)

Module-5

9 a. Solve : $(2x + y + 1)\,dx + (x + 2y + 1)\,dy = 0$. (07 Marks)

b. Solve : $(4xy + 3y^2 - x)\,dx + (x^2 + 2xy)\,dy = 0$. (06 Marks)

c. Solve : $y(2xy + e^x)\,dx - e^x\,dy = 0$. (07 Marks)

OR

10 a. Solve : $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (07 Marks)

b. Solve : $y(2xy + 1)dx - x\,dy = 0$. (06 Marks)

c. Solve : $\frac{dy}{dx} + y\cot x = \cos x$. (07 Marks)

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